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Mechanisms for the hard bubbling transition in symmetrically coupled chaotic systems

Sang-Yoon Kim^{1,2} and Woochang Lim²

¹ Institute for Research in Electronics and Applied Physics, University of Maryland, College Park, Maryland 20742, USA

² Department of Physics, Kangwon National University, Chunchon, Kangwon-Do 200-701, Korea

E-mail: sykim@kangwon.ac.kr

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Abstract

We investigate mechanisms for the hard bubbling transition in symmetrically coupled one-dimensional maps. A transition from strong to weak synchronization occurs via a first period-doubling or pitchfork transverse bifurcation of a periodic saddle embedded in the synchronous chaotic attractor. The consequence of such transverse bifurcations depends on the existence of an ‘absorbing area,’ controlling the global dynamics. In the presence of an absorbing area, a subcritical transverse bifurcation is found to give rise to abrupt appearance of transient intermittent bursts with large amplitude. For this case, any small parameter mismatch that is unavoidable in real systems induces a hard bubbling transition via an interior crisis. Through a detailed numerical analysis, we present explicit examples for the subcritical period-doubling and pitchfork transverse bifurcations, leading to hard bubbling.

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1. Introduction

Recently, the phenomenon of synchronization in coupled chaotic systems has become a field of intensive study. For this case of chaos synchronization, a synchronous chaotic motion may occur on an invariant subspace of the whole phase space [1–4]. Particularly, this chaotic synchronization has attracted much attention, because of its potential practical applications (e.g., see [5]).

An important problem in this field concerns stability of chaos synchronization with respect to a perturbation transverse to the invariant subspace [6]. If a synchronous chaotic state on the invariant subspace is transversely stable, then it may become an attractor in the whole phase space. Properties of transverse stability of the synchronous chaotic attractor (SCA) are intimately associated with transverse bifurcations of periodic saddles embedded in the

SCA [7–14]. If all such periodic saddles are transversely stable, then the SCA becomes asymptotically stable (i.e., Lyapunov stable and attracting in the topological sense), and hence we have ‘strong’ synchronization. However, as the coupling parameter passes through a threshold value, a periodic saddle first becomes transversely unstable through a local bifurcation. After this first transverse bifurcation, trajectories may be locally repelled from the invariant subspace when they visit the neighbourhood of the transversely unstable periodic repeller. Thus, loss of strong synchronization begins with such a first transverse bifurcation of an embedded periodic saddle, and then we have ‘weak’ synchronization. However, the fate of locally repelled trajectories through the first transverse bifurcation depends on whether there exists an absorbing area, surrounding the SCA and controlling the global dynamics, inside the basin of attraction [10–13, 15]. Particularly, for the case of asymmetric coupling the consequence of a transcritical transverse bifurcation has been well investigated in connection with existence of an absorbing area (see [12, 13]).

Here we are interested in symmetrically coupled chaotic systems, where transition from strong to weak synchronization may occur through a first period-doubling (PD) or pitchfork (PF) transverse bifurcation of a periodic saddle embedded in the SCA. For the case of a supercritical transverse bifurcation, it was found in [9, 10] that the unstable manifold of the newly born asynchronous saddle forms an absorbing area, surrounding the SCA and acting as a bounded trapping vessel. Then, a locally repelled trajectory from the invariant subspace is restrained within this absorbing area and exhibits a transient intermittent bursting from the invariant subspace [16]. For this case the ‘transverse’ size of the absorbing area increases gradually from zero with respect to variation of a control parameter, because the newly born asynchronous saddle points move away continuously from the invariant subspace. Hence, a supercritical transverse bifurcation induces a soft *bubbling transition* in the presence of parameter mismatch between the subsystems, because the maximum bursting amplitude increases gradually from zero. On the other hand, it was demonstrated in [8, 10] that a subcritical PD or PF transverse bifurcation may induce a *riddling transition*. For this case, the basin of the SCA becomes riddled with a dense set of ‘holes,’ belonging to the basin of another attractor (or infinity) [17]. This kind of riddling transition occurs when a synchronous saddle becomes transversely unstable by absorbing an asynchronous repeller lying at the basin boundary of the SCA. Before the riddling transition, the unstable manifold of the asynchronous repeller forms an absorbing area surrounding the SCA. However, at the moment of the riddling transition such an absorbing area disappears, because the SCA makes a contact with its basin boundary at the synchronous saddle points. Hence, a subcritical transverse bifurcation may induce a riddling transition in the absence of an absorbing area, because locally repelled trajectories may go to another attractor (or infinity). In the presence of parameter mismatch destroying the symmetric invariant subspace, the SCA with a riddled basin is transformed into a chaotic transient with a finite lifetime through a boundary crisis [18]. In contrast to this riddling transition, a subcritical PD or PF transverse bifurcation may also lead to an abrupt occurrence of intermittent burstings with large amplitude when an absorbing area, confining locally repelled trajectories, exists. However, dynamical origin for this hard bubbling transition in symmetrically coupled systems has been less understood than those for the case of asymmetric coupling [12, 13], although characterization of such intermittent burstings for both cases of the symmetric and asymmetric couplings has been well made in terms of the average interburst intervals [19].

In this paper, we investigate dynamical mechanisms for hard bubbling transition in a representative model system of symmetrically coupled one-dimensional (1D) maps. Unlike the case of riddling transition, a subcritical PD or PF transverse bifurcation may take place by absorbing a repeller lying strictly inside an absorbing area. For this case, the original absorbing

area, surrounding the SCA, is preserved through the subcritical transverse bifurcation, because there is no contact between the SCA and its basin boundary. In section 2.1, with explicit examples obtained through detailed numerical analysis, we discuss the consequence of subcritical PD and PF transverse bifurcations when an absorbing area, surrounding the SCA, exists. Just after this subcritical transverse bifurcation, a locally repelled trajectory is restricted to move within the absorbing area with a finite transverse width, and hence intermittent bursts with large amplitude appear abruptly. For this case, the SCA is still transversely stable, because its transverse Lyapunov exponent is negative. Hence the intermittent bursting will tend to stop. However, a small parameter mismatch between the 1D maps results in a continual sequence of intermittent hard bursts through an interior crisis, as shown in section 2.2. Thus, a mismatch-induced hard bubbling transition may occur in the presence of an absorbing area. Finally, a summary is given in section 3.

2. Hard bubbling transition in symmetrically coupled 1D maps

In this section, we investigate mechanisms for hard bubbling transition in symmetrically coupled 1D maps. In section 2.1, it is explicitly shown that, as a consequence of subcritical PD and PF transverse bifurcations, a transient intermittent bursting with large amplitude occurs abruptly when an absorbing area, surrounding the SCA, exists. Furthermore, in section 2.2 any small parameter mismatch that destroys the invariant diagonal is found to lead to a persistent intermittent hard bursting through an interior crisis.

2.1. Consequence of subcritical transverse bifurcations in the presence of an absorbing area

We consider two coupled 1D maps T ,

$$T: \begin{cases} x_{t+1} = f(x_t, a) + c(y_t - x_t) \\ y_{t+1} = f(y_t, b) + c(x_t - y_t) \end{cases} \quad (1)$$

where x_t and y_t are state variables of the subsystems at a discrete time t , the uncoupled dynamics ($c = 0$) is governed by the 1D map $f(x, p) = 1 - px^2$ with a control parameter p ($p = a, b$), and c is a coupling parameter. In a real situation, a small mismatch between the two 1D maps is unavoidable. To take into consideration this mismatch, we introduce a small mismatching parameter ε in the coupled 1D maps of equation (1) such that

$$b = a + \varepsilon. \quad (2)$$

In this subsection, we consider the ideal case without parameter mismatch ($\varepsilon = 0$). Then, the coupled map T has an exchange symmetry because it is invariant under the interchange of coordinates $x \leftrightarrow y$. The set of points which are invariant under the exchange operation forms a symmetric invariant line $x = y$. If an orbit lies on the symmetric invariant diagonal, it is called a synchronous orbit because the two state variables x_t and y_t become the same for all t ; otherwise, it is called an asynchronous orbit. In addition, this coupled map T is non-invertible, because its Jacobian determinant $\det(DT)$ (DT is the Jacobian matrix of T) becomes zero along the critical curve, $L_0 = \{(x, y) \in \mathbb{R}^2 : (2ax + c)(2by + c) - c^2 = 0\}$. Critical curves of rank k , L_k ($k = 1, 2, \dots$), are then given by the images of L_0 [i.e., $L_k = T^k(L_0)$]. Segments of these critical curves can be used to bound a compact region of the phase space that acts as a trapping bounded vessel, called an absorbing area \mathcal{A} , inside which trajectories bursting away from the diagonal are confined [15]. Furthermore, the boundary of an absorbing area can also be obtained by the union of segments of critical curves and portions of the unstable manifold of an unstable periodic orbit. For this case, \mathcal{A} is called a mixed absorbing area.

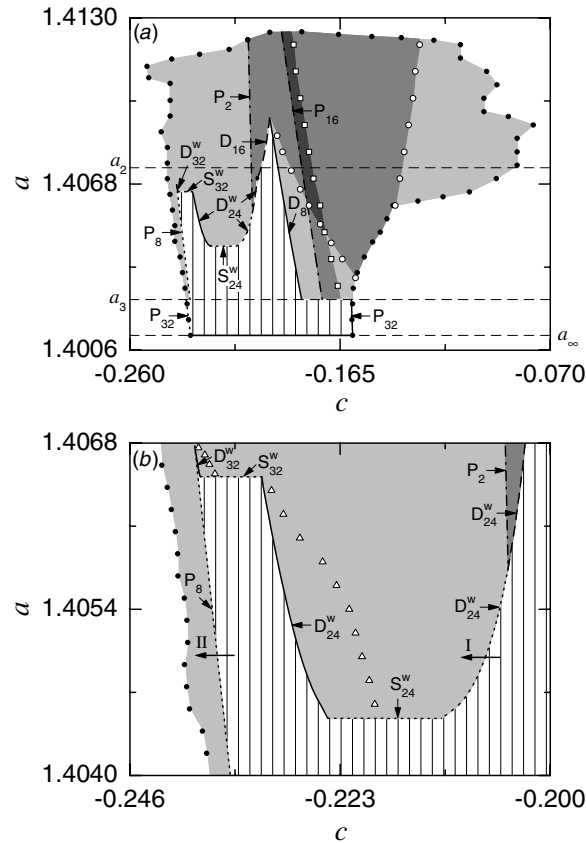


Figure 1. (a) Phase diagram in the second largest region of chaos synchronization in symmetrically coupled 1D maps. The SCA appears when crossing the critical line, denoted by the horizontal solid line on the $a = a_\infty$ line. Note that the SCA is strongly stable in the hatched region with vertical lines. The non-horizontal boundary curves D_q and P_q of the hatched region correspond to the first PD and PF transverse bifurcation curves of saddles with period q embedded in the SCA, respectively. (The dashed parts of D_{24}^w and D_{16}^w connect at a point, denoted by a small solid circle.) On the other hand, the horizontal boundary curve S_q represents the synchronous saddle-node bifurcation opening a period- q window. Here the superscript 'w' in S_q or D_q denotes that the bifurcation is associated with a periodic window. When passing this boundary curve, a bubbling or riddling transition occurs. The bubbling and riddling regions are shown in light grey and grey, respectively. Finally, the weakly stable SCA becomes transversely unstable via a blowout bifurcation when passing the boundary curve denoted by solid circles. (b) Magnified phase diagram of (a) near the D_{24}^w curve. For more details, see the text.

Based on the phase diagram for chaos synchronization, we first discuss the overall transverse stability of the SCA in the symmetrically coupled 1D maps of equation (1). With increasing control parameter a , the coupled map T exhibits an infinite sequence of PD bifurcations of synchronous attractors with period 2^n ($n = 0, 1, 2, \dots$), ending at the accumulation point $a_\infty (= 1.401155 \dots)$, in some region of c . When crossing a critical line in the a - c plane, a transition from periodic to chaotic synchronization occurs. Figure 1 shows the phase diagram in the second largest region of chaos synchronization. The SCA appears when crossing the critical line, denoted by the horizontal solid line on the $a = a_\infty$ line. With further increase of a from a_∞ , a sequence of band-merging bifurcations of the SCA takes

place. For $a = a_n$, the 2^{n+1} bands of the SCA merge into 2^n bands; $a = a_2 (= 1.407405 \dots)$ and $a = a_3 (= 1.402492 \dots)$ lines are shown in the figure.

For the chaotic values of a , the SCA is at least weakly stable inside the region bounded by solid circles in figure 1, because its transverse Lyapunov exponent

$$\sigma_{\perp} = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{t=1}^N \ln |2ax_t + 2c| \quad (3)$$

is negative. We note that the SCA becomes strongly stable in the hatched region with vertical lines, because there all periodic saddles embedded in the SCA are transversely stable. When crossing the boundary of this hatched region, a weakly stable SCA exhibiting bubbling or riddling appears, depending on the existence of an absorbing area. For the bubbling case, there exists an absorbing area, restraining the bursting from the diagonal. However, in the riddling case, there exists no such absorbing area, and hence the basin of the SCA becomes riddled with a dense set of holes, belonging to the basin of another attractor (or infinity). Note that these bubbling and riddling regions are shown in light grey and grey, respectively. In this region of weak synchronization, a transition from bubbling to riddling may occur through a boundary crisis of an absorbing area [10] when passing a curve denoted by open circles in figure 1(a) or through stabilization of an asynchronous saddle with period q [9] when crossing a dash-dotted PF bifurcation curve P_q . Thus, in the dark grey region, the basin of the SCA becomes riddled with two dense sets of holes, belonging to the basin of the attractor at infinity and to the basin of a stabilized asynchronous attractor inside the absorbing area. The asynchronous chaotic attractor developed from the stabilized asynchronous periodic attractor disappears through a boundary crisis when crossing a curve denoted by open squares in figure 1(a). In addition, transition from a small to a large absorbing area may occur via an interior crisis when passing the curve denoted by open triangles in figure 1(b), and then the maximum bursting amplitude increases abruptly. Finally, when crossing the boundary denoted by solid circles, the SCA becomes transversely unstable through a blowout bifurcation [20], and then a complete desynchronization occurs.

The non-horizontal boundary curves D_q and P_q of the hatched region with vertical lines represent the first PD and PF transverse bifurcations of a period- q saddle embedded inside the SCA, which occur when its transverse Floquet (stability) multiplier,

$$\lambda_{\perp} = \prod_{t=1}^q (-2ax_t - 2c) \quad (4)$$

passes through -1 and $+1$, respectively. Note that the consequence of these transverse PD and PF bifurcations depends on the existence of an absorbing area. When crossing a solid curve (e.g., see the solid part of the D_{24}^w curve), a soft bubbling transition occurs through a supercritical transverse bifurcation [9, 10], because the unstable manifold of a newly born asynchronous saddle forms a mixed absorbing area, inside which burstings from the diagonal are restrained. Then, the maximum bursting amplitude increases gradually from zero. On the other hand, when crossing a dashed curve (e.g., see the dashed part of the D_{24}^w curve), a riddling transition takes place through a subcritical transverse bifurcation [8, 10] by absorbing an asynchronous repeller at the basin boundary of the SCA. For this case, an absorbing area, surrounding the SCA, disappears, because the SCA makes a contact with its basin boundary. Then, the basin of the SCA becomes riddled with a dense set of holes, belonging to the basin of another attractor (or infinity). Thus, a subcritical transverse bifurcation may induce a riddling transition in the absence of an absorbing area. However, when passing a dotted curve (e.g., see the dotted part of the D_{24}^w curve), a hard bubbling transition occurs through a subcritical

transverse bifurcation in the presence of an absorbing area, as demonstrated below. For this case, intermittent bursts with large amplitude appear abruptly.

Through a detailed numerical analysis, we investigate the consequence of subcritical PD and PF transverse bifurcations when an absorbing area, surrounding the SCA, exists. As a first example, consider the subcritical PD bifurcation of a synchronous period-24 saddle (born via a saddle-node bifurcation) that occurs when crossing the dotted part of the D_{24}^w curve in figure 1(a). Figure 1(b) shows a magnified phase diagram of figure 1(a) near the D_{24}^w curve. An asynchronous period-2 saddle becomes stabilized when crossing the dash-dotted subcritical PF bifurcation line P_2 , and then the basin of the SCA becomes riddled with a dense set of holes, belonging to the basin of the stabilized asynchronous period-2 attractor. Thus, a transition from bubbling (shown in light grey) to riddling (shown in grey) occurs. Note that this stabilization line P_2 touches the right subcritical part of the D_{24}^w curve and decomposes it into the dashed and dotted parts. When crossing the dashed part, the subcritical PD bifurcation induces a riddling transition, because an absorbing area, surrounding the SCA, disappears at the bifurcation point. On the other hand, when passing the dotted part, transient intermittent bursts with large amplitude appear suddenly, because there exists an absorbing area. For explicit demonstration, we fix the value of a as $a = 1.405$ and decrease the coupling parameter c along the route I in figure 1(b). Figure 2(a) shows a large absorbing area surrounding the whole eight-band SCA for $c = -0.202$. Inside this absorbing area, an asynchronous period-48 repeller (denoted by solid circles) lies near a synchronous period-24 saddle (denoted by open circles) embedded in the SCA. For a clear view, a magnified view near the topmost band of the SCA is given in figure 2(b). As c is decreased, the asynchronous period-48 repeller approaches the synchronous period-24 saddle. Eventually, when crossing the dotted part of the D_{24}^w curve for $c = -0.207085$, the synchronous period-24 saddle becomes transversely unstable via a subcritical PD bifurcation by absorbing the asynchronous period-48 repeller lying inside the absorbing area. For this case, the absorbing area surrounding the SCA is preserved, as shown in figure 2(c) for $c = -0.212$, in contrast to the case of riddling transition. Thus, transient intermittent bursts with large amplitude appear abruptly (see figure 2(d)). For this case, the maximum bursting amplitude is determined by the transverse size of the absorbing area.

As a second example, we consider the subcritical PF transverse bifurcation that occurs when crossing the dotted P_8 line in figure 1(b). For explicit demonstration, we fix the value of a as $a = 1.405$ and decrease the coupling parameter along the route II in figure 1(b). Figure 3(a) shows a large absorbing area surrounding the whole eight-band SCA for $c = -0.233$. Inside this absorbing area, a pair of asynchronous period-8 repellers (denoted by solid up-triangles and down-triangles) lies near a synchronous period-8 saddle (denoted by open circles). To show the repellers clearly, a magnified view near the topmost band of the SCA is given in figure 3(b). As c is decreased, the asynchronous period-8 repellers approach the synchronous period-8 saddle. Eventually, for $c = -0.236431$ the synchronous period-8 saddle loses its transverse stability via a subcritical PF bifurcation by absorbing the asynchronous period-8 repellers. For this case, the large absorbing area is preserved, as shown in figure 3(c) for $c = -0.238$. Thus, a transient intermittent bursting with large amplitude appears suddenly (see figure 3(d)), as in the above case of the subcritical PD transverse bifurcation.

As explicitly demonstrated in the above two examples, subcritical PD and PF transverse bifurcations give rise to an abrupt appearance of transient intermittent bursts with large amplitude when there exists an absorbing area, surrounding the SCA. For this case, any small parameter mismatch between the 1D maps may lead to a persistent intermittent bursting through an interior crisis, as will be shown in section 2.2.

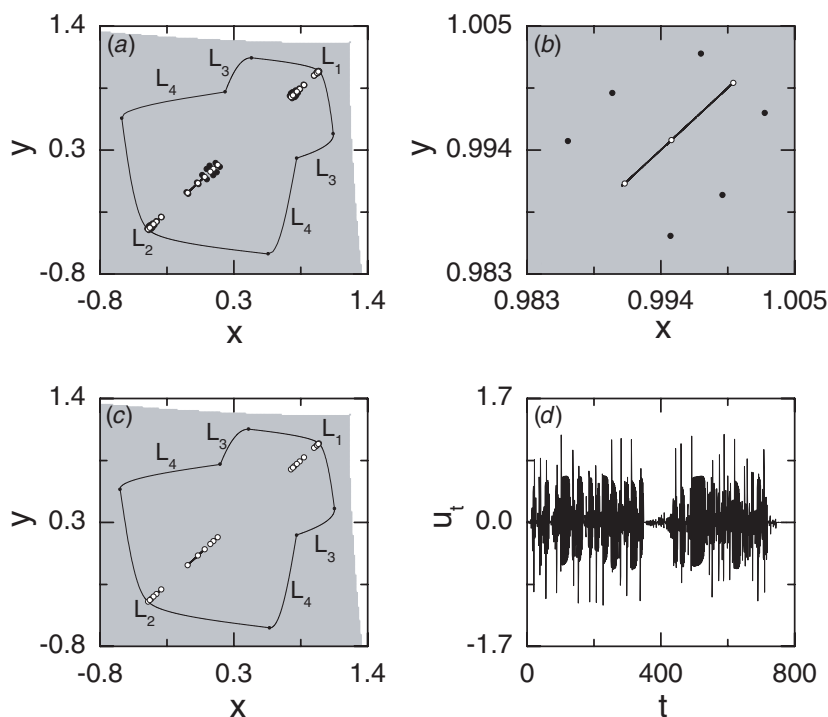


Figure 2. (a) Large absorbing area surrounding the whole eight-band SCA for $a = 1.405$ and $c = -0.202$ before the subcritical PD transverse bifurcation of the embedded synchronous period-24 saddle (denoted by open circles). The absorbing area is bounded by segments of the critical curves L_k ($k = 1, \dots, 4$) inside the basin of attraction (shown in light grey). Here the dots indicate where the segments of the critical curves connect. An asynchronous period-48 repeller (denoted by solid circles) lies strictly inside the absorbing area. For a clear view of this asynchronous period-48 repeller, a magnified view of (a) near the topmost band of the SCA is given in (b). For $c = -0.207085$, the synchronous period-24 saddle becomes transversely unstable via a subcritical PD bifurcation by absorbing the asynchronous period-48 repeller. (c) Preserved absorbing area surrounding the SCA for $a = 1.405$ and $c = -0.212$ after the subcritical PD transverse bifurcation. (d) Transient intermittent bursting for a trajectory starting from an initial point $(x_0, y_0) = (0.5, 0.51)$ when $a = 1.405$ and $c = -0.212$. Here the transverse variable u ($\equiv y - x$) represents the deviation from the diagonal.

2.2. Effect of parameter mismatch on transient intermittent bursting

In this subsection, we investigate the parameter-mismatching effect on the transient intermittent bursts caused by subcritical PD and PF transverse bifurcations in the presence of an absorbing area. It is thus shown that any small parameter mismatch leads to a persistent intermittent hard bursting via an interior crisis.

We first consider the case of a subcritical PD transverse bifurcation discussed in section 2.1 and study the effect of parameter mismatch with $\varepsilon = 0.0015$ along the route I for $a = 1.405$ in figure 1(b). For this case, a strongly stable SCA has no sensitivity with respect to variation of the mismatching parameter, because all embedded unstable periodic orbits are transversely stable [19]. Hence, the strongly stable SCA in figure 2(b) is transformed into a thin ‘quasisynchronous’ chaotic attractor that lies in a slightly spread invariant region near the diagonal. For a clear presentation, a magnified view of the quasisynchronous chaotic attractor near its topmost band is given in figure 4(a) for $c = -0.202$. A period-48 repeller

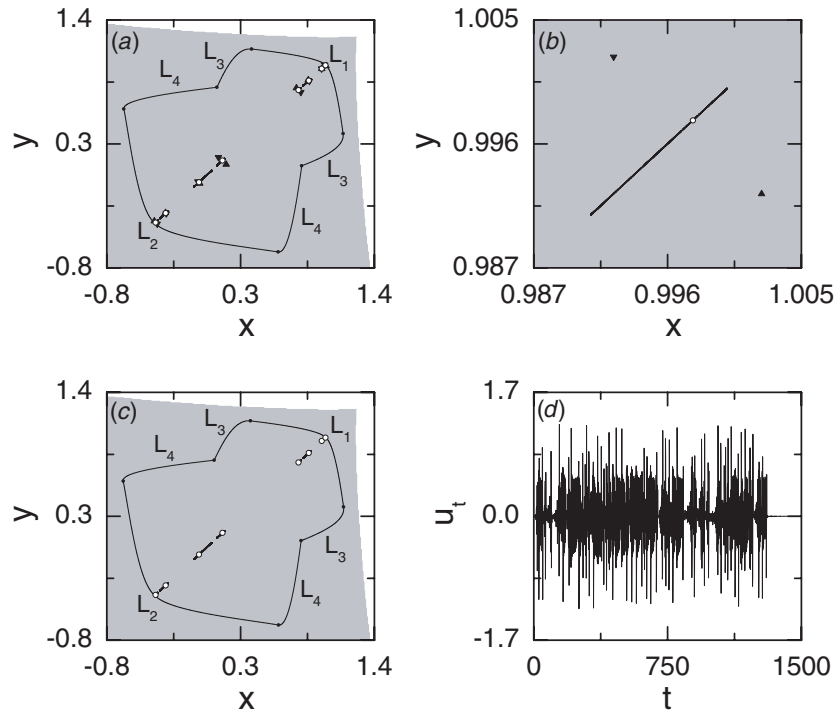


Figure 3. (a) Large absorbing area surrounding the whole eight-band SCA for $a = 1.405$ and $c = -0.233$ before the subcritical PF transverse bifurcation of the synchronous period-8 saddle (denoted by open circles). The absorbing area is bounded by segments of the critical curves L_k ($k = 1, \dots, 4$) inside the basin of attraction (shown in light grey). Here the dots indicate where the segments of the critical curves connect. A pair of asynchronous period-8 repellers (denoted by solid up-triangles and down-triangles) lies strictly inside the absorbing area. For a clear view of the asynchronous period-8 repellers, a magnified view of (a) near the topmost band of the SCA is given in (b). For $c = -0.236431$, the synchronous period-8 saddle becomes transversely unstable via a subcritical PF by absorbing a pair of asynchronous period-8 repellers. (c) Preserved absorbing area surrounding the SCA for $a = 1.405$ and $c = -0.238$ after the subcritical PF transverse bifurcation. (d) Transient intermittent bursting for a trajectory starting from an initial point $(x_0, y_0) = (0.5, 0.51)$ when $a = 1.405$ and $c = -0.238$. Here the transverse variable u ($\equiv y - x$) represents the deviation from the diagonal.

(denoted by solid circles) lies near a period-24 saddle (denoted by open circles) embedded inside the quasisynchronous chaotic attractor. As c is decreased the period-48 repeller approaches the quasisynchronous chaotic attractor. Eventually, when passing a threshold value of $c = -0.2069$ they make a collision, and then a large two-dimensional (2D) chaotic attractor filling the whole absorbing area appears suddenly via an interior crisis, as shown in figure 4(b) for $c = -0.212$. Consequently, a persistent intermittent bursting with large amplitude occurs abruptly (see figure 4(c)). After this interior crisis, a subcritical PD bifurcation between the period-24 saddle and the period-48 repeller takes place for $c = -0.206950$ inside the large 2D chaotic attractor. Although the PD bifurcation is preserved under a parameter mismatch, it has no particular consequence to the hard bubbling caused by the interior crisis.

As a second example, we consider the case of a subcritical PF transverse bifurcation discussed in section 2.1 and investigate the effect of parameter mismatch along the route II

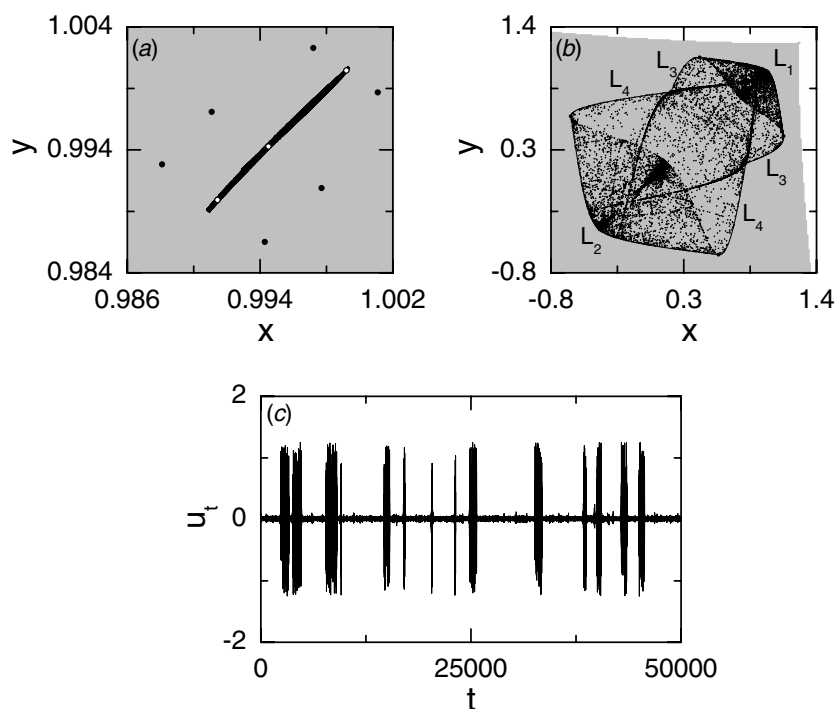


Figure 4. Effect of the parameter mismatch with $\varepsilon = 0.0015$ on the strong and weak synchronization for $a = 1.405$ in the case of the subcritical PD transverse bifurcation. (a) A magnified view near the topmost band of the thin quasisynchronous chaotic attractor for $c = -0.202$ inside the basin of attraction shown in grey. Solid and open circles represent the period-48 repeller and period-24 saddle, respectively. (b) A large 2D chaotic attractor filling the whole absorbing area and (c) a persistent intermittent bursting for a trajectory starting from $(x_0, y_0) = (0.5, 0.5)$ for the hard bubbling case of $c = -0.212$. Here the transverse variable u ($\equiv y - x$) represents the deviation from the diagonal.

for $a = 1.405$ in figure 1(b). Unlike the case of the PD bifurcation, the PF bifurcation is structurally unstable, and hence it is destroyed even by a small mismatch. Figure 5(a) shows the bifurcation diagram for $\varepsilon = 0.0002$. Note that the subcritical PF bifurcation is transformed into a saddle-node bifurcation through splitting of the upper branch of the original bifurcation diagram for $\varepsilon = 0$. Hence the period-8 saddle (denoted by the solid line) disappears through collision with a period-8 repeller (denoted by the dashed curve). As in the above case of the subcritical PD transverse bifurcation, a strongly stable SCA without parameter sensitivity is transformed into a thin quasisynchronous chaotic attractor in the presence of a parameter mismatch. Figure 5(b) shows a magnified view near the topmost band of the quasisynchronous chaotic attractor for $c = -0.233$. Two period-8 repellers (denoted by the up-triangle and down-triangle) approach the quasisynchronous chaotic attractor, and a period-8 saddle (denoted by an open circle) lies at the upper boundary of this quasisynchronous chaotic attractor. When passing a threshold value of $c = -0.2345$, the upper period-8 repeller makes a collision with the period-8 saddle in a saddle-node bifurcation, and then they disappear. Consequently, the quasisynchronous chaotic attractor is transformed into a large 2D chaotic attractor filling the whole absorbing area, due to the interior crisis induced by the saddle-node bifurcation, as shown in figure 5(c) for $c = -0.238$. Thus, a persistent intermittent bursting with large amplitude occurs abruptly (see figure 5(d)).

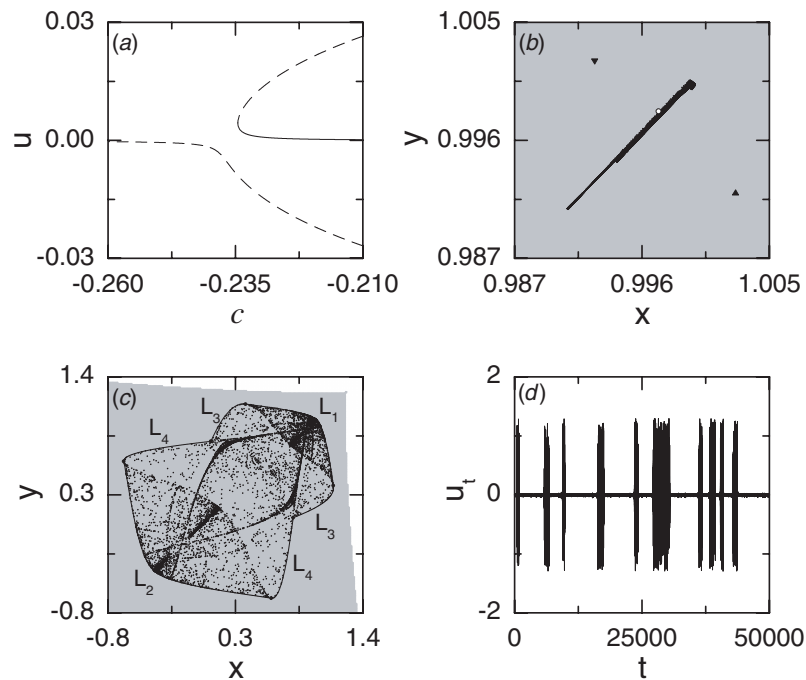


Figure 5. Effect of the parameter mismatch with $\varepsilon = 0.0002$ on the strong and weak synchronization for $a = 1.405$ in the case of the subcritical PF transverse bifurcation. (a) Bifurcation diagram. Solid and dashed lines represent the period-8 saddle and repeller, respectively. (b) A magnified view near the topmost band of the thin quasiperiodic chaotic attractor for $c = -0.233$ inside the basin of attraction shown in grey. The up-triangle and down-triangle denote period-8 repellers, while the open circle represents the period-8 saddle. (c) A large 2D chaotic attractor filling the whole absorbing area and (d) a persistent intermittent bursting for a trajectory starting from $(x_0, y_0) = (0.5, 0.5)$ for the hard bubbling case of $c = -0.238$. Here the transverse variable u ($\equiv y - x$) represents the deviation from the diagonal.

We also note that equation (1) is invariant under the interchange $x \leftrightarrow y$ and $\varepsilon \mapsto -\varepsilon$. Hence, the same kind of hard bubbling transition occurs through an interior crisis, independently of the sign of parameter mismatch.

3. Summary

In connection with existence of an absorbing area controlling the global dynamics, we have investigated some mechanisms for hard bubbling transition in symmetrically coupled 1D maps. Through a detailed numerical analysis, we have explicitly demonstrated that subcritical PD and PF transverse bifurcations may lead to an abrupt occurrence of a transient intermittent bursting with large amplitude in the presence of an absorbing area. For this case, any small parameter mismatch that is unavoidable in real systems has been shown to induce a hard bubbling transition via an interior crisis.

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